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Effects of variable viscosity and thermal conductivity on MHD flow past a vertical plate

G. C. Hazarika
Dibrugarh University

Utpal Sarma Gopal Ch.
Joya Gogoi College

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Abstract

Heat and mass transfer flow along a vertical plate under the combined buoyancy force of thermal and species diffusion in the presence of a transverse magnetic field is investigated.
The boundary layer equations are transformed in to ordinary differential equations with similarity transformations. The effects of variable viscosity and thermal conductivity on velocity profile, temperature profile and concentration profiles are investigated by solving the governing transformed ordinary differential equations with the help of Runge-Kutta shooting method and plotted graphically.

Keywords: Variable viscosity, thermal conductivity, magnetic field.

MSC(2000): 47.10AD

1 Introduction

Many transport processes occur in industrial applications in which the trans fer of heat and mass takes place simultaneously as a result combined buoyancy effects of thermal diffusion and diffusion of chemical species. Many studies have been reported for vertical, horizontal and inclined plate in presence of a transverse magnetic field. The problem was solved by Elbashbeshy [1] without the mass transfer effect, variable viscosity and thermal con ductivity. In this study the induced magnetic field is neglected. Soundalgekar [2] studied the effects of mass transfer on free convective flow of a dissipative incompressible fluid past an infinite vertical porous plate with suction. Bhadauria [3] also studied time periodic heating of Reyleigh Benard convection in a vertical magnetic field. Mostafa Mohmoud [4] has studied the variable viscosity and chemical reaction effects on mixed convection heat and mass transfer along a semi infinite vertical plate. Singh [5] presented paper on unsteady hydromagnetic free con vective flow past a vertical infinite flat plate. Recently Kafousias [6] presented an analysis of the effect of temperature dependent viscosity on free convective laminar boundary layer flow past a vertical isothermal flat plate. Ganesan and Palani [7] studied numerical solution of transient free convection MHD flow of an incompressible viscous fluid flow past a semi infinite inclined flat plate with variable surface heat and mass flux. The set of governing equations are solved using implicit finite difference scheme. Kafousias [8] has studied the heat and mass transfer along a vertical plate in the presence of a magnetic field. Pantokratoras [9] has studied

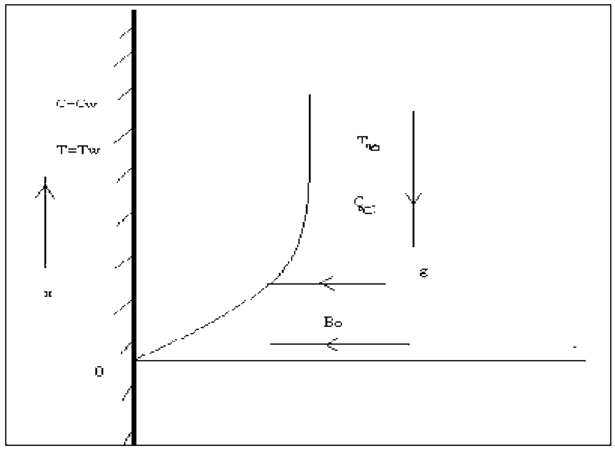

Figure 1: Some relevant variables by the mathematical formulation of the problem

the free convection along a vertical, isothermal plate under the effect of a constant, horizontal, magnetic field. Takhar and Soundalgekar [10] have studied the dissipation effects on MHD free convection flow past a semi-infinite vertical plate. Sparrow and Cess [11] presented their research work on the effect of a magnetic field on free convection heat transfer.

In most of the studies, of this type of problems, the viscosity and thermal conductivity of the fluid were assumed to be constant. However, it is known that these physicsl properties can change significantly with temperature and when the effects of variable viscosity and thermal conductivity are taken in to account, the flow characteristics are significantly changed compared to constant property case. Hence the problem under consideration, the viscosity and thermal conductivity have been assumed to be inverse linear function of temperature.

2 Mathematical formulation of the problem

We are considering here an incompressible flow past a vertical plate along the axis of x and y-axis is perpendicular to it. The plate is along the direction of xaxis. Here Twand Cware temperature and concentration of the fluid at the plate and T1 and C1 are the temperature and concentration outside the boundary layer.

Since the velocity of the fluid is low the viscous dissipative heat is assumed to be negligible. A magnetic is field of constant intensity Bois applied in a direction transverse to the plate and the elecrical conductivity of the fluid is assumed to be small so that the induced magnetic field can be neglected in comparison to applied magnetic field, g is the direction of gravitational force vertically downwd as shown in Figure 1

The flow governing equations now can be written as

$$\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0 \tag{1}$$

$$u'\frac{\partial u'}{\partial x'}+v'\frac{\partial u'}{\partial y'}=\nu\frac{\partial^2 u'}{\partial y'^2}+\frac{\partial u'}{\partial y'}\frac{\partial \nu'}{\partial y'}+g\beta(T'-T_\infty)+g\beta'(C'-C_\infty)-\frac{\sigma B^2}{\rho}u' \tag{2}$$

$$u'\frac{\partial T'}{\partial x'}+v'\frac{\partial T'}{\partial y'}=\alpha\frac{\partial^2 T'}{\partial y'^2}+\frac{1}{\rho_{\alpha C_p}}\frac{\partial T'}{\partial y'}\frac{\partial k}{\partial y'} \tag{3}$$

$$u'\frac{\partial C'}{\partial x'}+v'\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}+\frac{\partial C'}{\partial y'}\frac{\partial \nu}{\partial y'}. \tag{4}$$

where C' is the concentration, T' is the temperature, $(\beta,\beta^*\ )$ are the temperature and concentration coeffcients of volumetric expansion, ν is the kinematic viscosity, the velocities are (u',v$'$) along the axes (x',y'). The equations (2)-(4) must be solved subject to the boundary conditions: At

$$y=0;\ u=v=0;\ T=T_w;\ C=C_w$$

As

$$y\to\infty; u=0; T=T_\infty; C=C_\infty. \tag{5}$$

The velocity components along the axes can be expressed as: $u=\frac{\partial\psi'}{\partial y'}$, $v=-\frac{\partial\psi'}{\partial x'}$ where $\psi$is the stream function such that the continuity equation is satised. $U=\sqrt{gl\beta(T_w-T_\infty)}$ is a quantity with the dimension of speed and $Gc=g\beta L^3(T_w-T_\infty)/\nu_\alpha^2$is the Grashof number, L is a typical length scale. Now we introduce the following non-dimenssional variables.

$$\psi'=x^{\frac{3}{4}}f(\eta),\ \eta=x^{-\frac{1}{4}}y,\ T=\theta(\eta),\ C=\phi(\eta)$$

$$x=\frac{x'}{l},\ y=\frac{x'}{l}Gc^{\frac{1}{4}},\ \psi=\psi'\frac{Gc^{\frac{1}{4}}}{UL},\ B^2=B_0x^{\frac{1}{4}}$$

$$\theta=\frac{T'-T_\infty}{T_w-T_\infty},\ \phi=\frac{C'-C_\infty}{C_w-C_\infty}. \tag{6}$$

Here θ and ϕ are non dimensional temperature and concentration. Viscosity and thermal conductivity are inverse lnear functions of temperature, following Lai and Kulacki [12], we assume,

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}[1+\gamma(T-T_\infty)] \tag{7}$$

$$\frac{1}{\mu}=b(T-T_\infty),\quad where\quad b=\frac{\gamma}{\mu_\infty},$$

and

$$T_c = T_\infty - \frac{1}{\gamma}.$$

Again

$$\frac{1}{k} = \frac{1}{k_\infty}[1 + k(T - T_\infty)], \tag{8}$$

or

$$\frac{1}{k} = \alpha(T - T_r), \quad where \quad \alpha = \frac{k}{k_\infty},$$

and

$$T_r = T_\infty - \frac{1}{k},$$

where α, β, T_c, T_r are constants and their values depend on the reference state and thermal properties of the fuid i.e. ν and k. In general b > 0 for liquids and b < 0 for gases. The non-dimensional form of viscosity and thermal conductivity parameters θ_c and θ_r can be written as,

$$\theta_c = \frac{T_c - T_\infty}{T_w - T_\infty} = \frac{1}{\gamma(T_w - T_\infty)} \tag{9}$$

$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} \tag{10}$$

Substituting equation (7)-(10) in to equations (2)-(4) we have

$$f''' = [\frac{3}{4}f'' - \frac{1}{2}f'^2 - Mf' + e\phi]\frac{\theta - \theta_c}{\theta_c} + \frac{\theta'}{\theta - \theta_c}f'' \tag{11}$$

$$\theta'' = pr\frac{\theta' - \theta_r}{\theta_r}[\frac{3}{4}f] + \frac{\theta'}{\theta - \theta_r} \tag{12}$$

$$\phi'' = Sc\phi'\frac{\theta - \theta_c}{\theta_c}[\frac{3}{4}] + \frac{\theta'}{\theta - \theta_c}\phi' f'' \tag{13}$$

The boundary conditions with the new variables are

$$\eta = 0,\ f = f'' = 0,\ \theta = \phi = 1$$
$$\eta \to \infty,\ f' = 0,\ \theta = \phi = 0 \tag{14}$$

Equations (10)-(12) with boundary conditions (13) describe the heat and mass transfer along a vertical plate in the presence of a magnetic field under variable viscosity and thermal conductivity. In equation (11) $e = \frac{\beta^*(C_w - C_\infty)}{\beta(T_w - T_\infty)}$ represents the relative effect of chemical diffusion on thermal diffusion. When e=0 there is no mass diffusion and the buoyancy force arises solely from the temperature difference. Here a prime denotes differentiation with respect toη. The governing equations (11)-(13) with boundary conditions given in (14) are solved numerically by using the 4th order Runge-Kutta Shooting method.

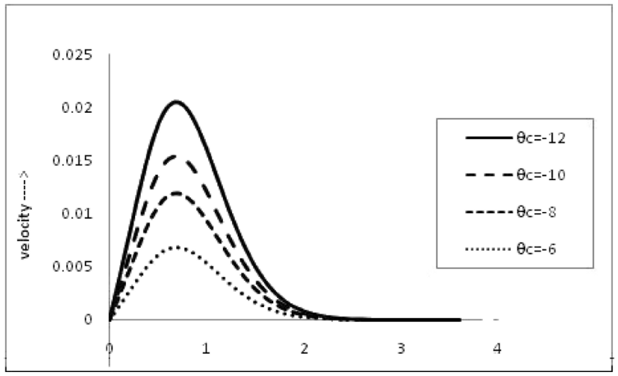

Figure 2: Velocity profile decreases with the increase of θ_c

3 Results and discussion

The system of differential equations (11)-(13) goverened by the boundary conditions are solved numerically by an effcient numerical technique based on the fourth order Runge-Kutta method Shooting method. The numerical method can be programmed and appplied easily. It is experienced that the convergence of the iteration process is quite rapid. Solutions were obtained for pr=.73 and various values of θ_c, θ_r, M, Ec and Sc respectively. The viscosity-temperature variation and conductivity-temperature variation are represented by the dimensionless parameters θ_c and θ_r respectively whereas the magnetic field and the concentration field effect are represented by the dimensionless parameters M and Sc respectively. When the temperature difference $\nabla T = T_w - T_\infty$ is positive as in our case the viscosity and thermal condctivity parameter θ_c and θ_r are negative for fluids and positive for gases [12]. The concept of variable viscosity was first introduced by Ling and Dybbs [13], on their study of forced convective flow in porous medium. The viscosity and thermal conductivity temperature equations can be written as $\mu = \frac{\mu_\infty}{1-\theta_c^{-1}}$ and $k = \frac{k_\infty}{1-\theta_r^{-1}}$. It is obvious from the above expressions that for physical reality θ_c and θ_r cannot take values 0 and 1. It is exerienced that when θ_c and θ_r are large, viscosity and thermal conductivity variation in the boundary layer is negligible, but when θ_c and $\theta_r \rightarrow 1$ the viscosity and thermal conductivity become increasingly signicant.

In Figure 2 we are substituting the values for different parameters like Prandtl number pr=0.73, magnetic field parameter M=0.1, ratio of thermal diffusivity to concentration diffusivity e=0.1, thermal conductivity parameter θ_r =-10. Substituting different values of the viscosity parameter θ_c we observe that the velocity profile decreases with the increase of θ_c. The effect of variable viscosity is not so prominent in case of temperature profile.

In Figure 3 we observe the effect of Schemidt number on concentration profile considering the valuies of Sc=3.1, 4.1, 5.1, 7.1, 9.1 with the values of the param-

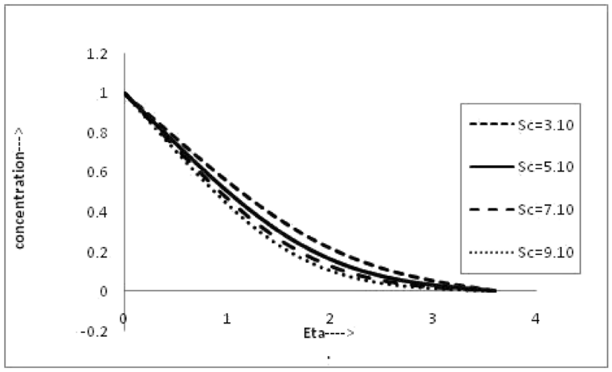

Figure 3: Effect of Sc on concentration

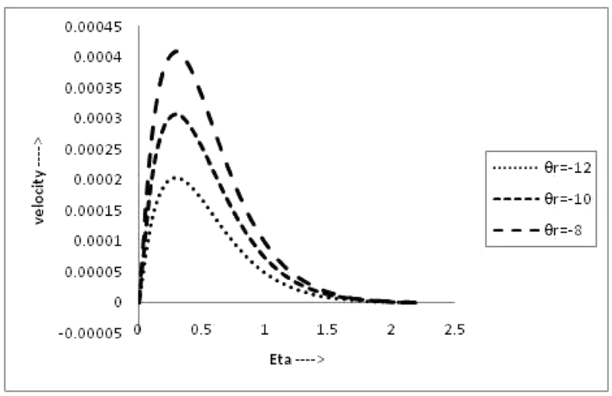

Figure 4: Variation of velocity with θ_r

eters pr=0.73, magnetic field parameter M=0.1, ratio of thermal diffusivity to concentration diffusivity e=0.1, thermal conductivity parameter θ_r =-10. A rise in Sc strongly suppresses the concentration levels in the boundary layer regime. All profiles decay monotonically from the surace to the free stream. Sc embodies the ratio of momentum diffusivity to molecular diusivity. It is conclude that an increase in Sc, the concentration decreases.

In Figure 4 we study the variations of the velocity profile for different values of the thermal conductivity parameter. Here we substitute the values of thermal conductivity parameter like θ_r = 10, -8 and other parameters M=.1, e=.1, Sc=1, θ_c =- 10; pr=.73 and finally observe that the velocity profile increases with the decreases of thermal conductivity parameter.

In Figure 6 we observe the effect of variable viscosity parameter on concentration profile. The values of the variables considered here are pr=.73, θ_r = -10, e=.1, Sc=1 and θ_c = -10, -3, -1. And it is observed that the concentration profile

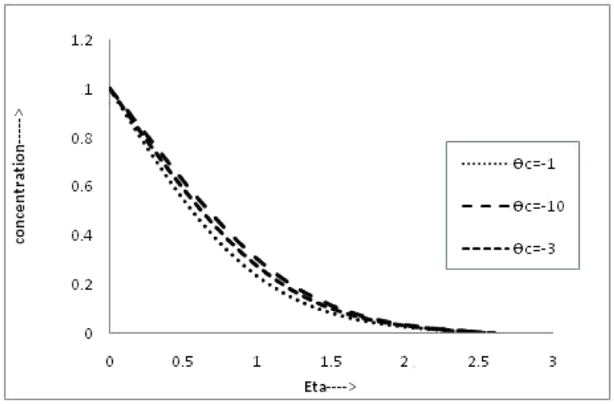

Figure 5: Variation of concentration with θ_c

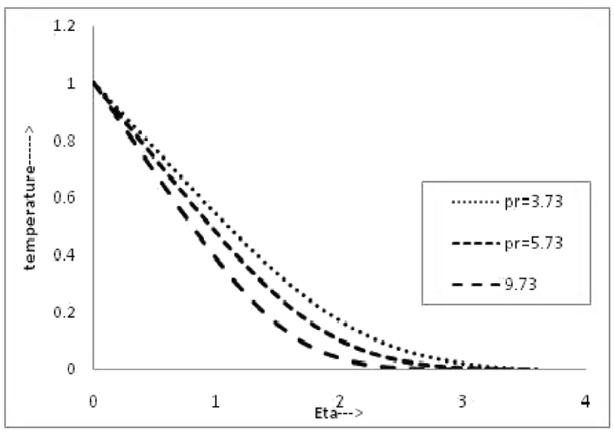

Figure 6: Variation of temperature with pr

decreases with the increase of variable viscosity.

In Figure 6 we study the effect of the temperature profile for various values of the Prandtl number. We assume the values of pr= 3.73, 5.73, 9.73. Also the other parameters we assume like θ_c = -10, θ_r = -10, M=.1, e=.1, Sc=1. And it is observed that the temperature profile decreases with the increase of Pr.

In Figure 7 we study the effect of magnetic field parameter M on the velocity profiles. We substitute the various values of the Hartmann number M= .3, .7, 1, 1.3 and the other values of the parameters has been considered as pr=.73, θ_c =- 10, θ_r = -10, e=.1, Sc=1. The velocity profiles decreases with the increase of Hartmann number M due to Lorentz force.

In Figure 8 we observe the effect of the magnetic field M on the concentration profile. Substituting various values of M=3.01, 5.31, 7.31, 9.91 and pr=.73, Sc=1,

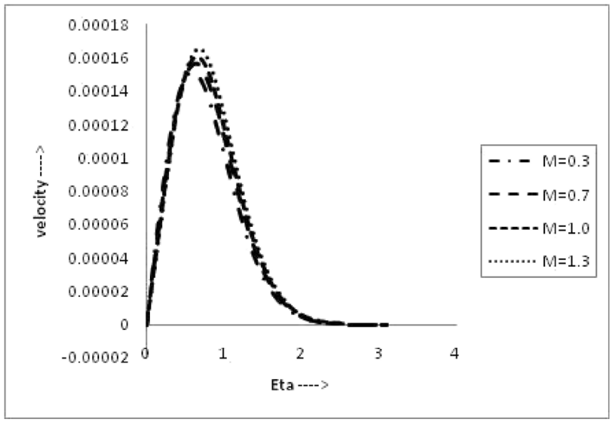

Figure 7: Variation of velocity with M

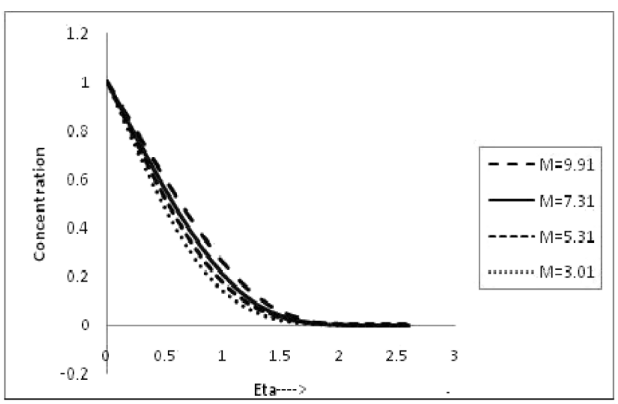

Figure 8: Variation of concentration with M

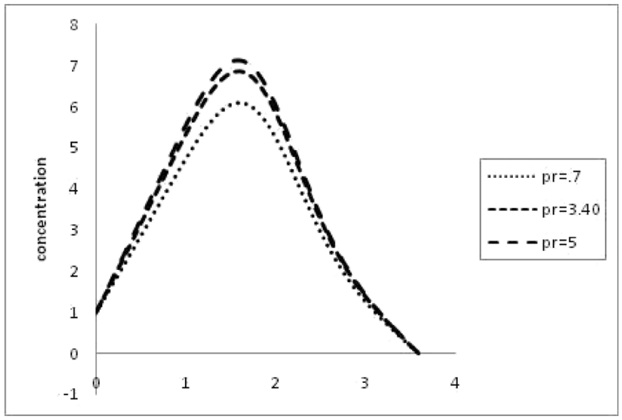

Figure 9: Effect of Prandtl number pr on concentration profile

$\theta_c =$ -10, $\theta_r =$-10; it is observed that the concentration profile decreases with the increase of M. Observation is very analogous with the theory because due to the transverse magnetic field a drag force is developed which opposes the flow. It has been observed that the concentration decreases with the increase of the magnetic field.

In Figure 9 we study the effect of the Prandtl number pr on the concentration profile. The study reveals that the concentration profile increases with the increase of pr.

References

[1] Elbashbesy, E. M. A., Heat and mass transfer along a vertical plate in the presence of a magnetic field, Indian J. pure and applied Math., 27(6), (1996), pp. 624-631.

[2] Soundalgekar, V. M., Effects of mass transfer on free convective flow of a dissipative incomplressible fluid past an infinite vertical porous plate with suction, proc. Indian Acad. Sci., 84A (5), (1976), pp. 194.

[3] Bhadauria, B. S., Time periodic heating of Rayleigh-Benard convection in a vertical magnetic field, J. Phys. Scr, 73, (2006), pp. 296-302.

[4] Mostafa, A. A., Mahmoud, Variable viscosity and chemical reaction effects on mixed convection heat and mass transfer along a semi infinite vertical plate, J. Mathematical problems in Engineering, Article ID 41323, (2007), 7 pages.

[5] Singh, D., Unsteady hydromagnetic free convection flow past a vertical infinite at plate, J. of physical society of Japan, Vol. 19, (1964), pp 751-755.

[6] Kafoussius, N. G. and Williams, E.W., The effect of temperature dependent viscosity on free convective laminar boundary layer flow past a vertical isothermal flat plate, Acta Mechanica, Vol. 110, No.1-4, (1995), pp 123-137.

[7] Ganesan, P. and Pilani, G., Numerical solution of unsteady MHD flow past a semi-infinite isothermal vertical plate, in proceedings of 6th ISHMT/ASME Heat and Mass transfer conference and 17th Heat and Mass transfer conference, (2004), pp 184-187.

[8] Kafousias, N. G., MHD thermal diffusion effects on free convective heat and mass transfer flow over an infinite vertical moving plate, Astrophysics,Space Sci., Vol. 92, (1992), pp 11-19.

[9] Pantokratoras, A., The effect of variable viscosity on mixed convection heat transfer along a vertical moving surface, Int. J. of Thermal Sciences, Vol. 45, (2006), pp60-69.

[10] Takhar, H. S. and Soundalgekar, V.M., Dissipation effects on MHD free convection flow past a semi infinite vertical plate, App. Scientic Research, Vol. 36, (1980), pp 163-171.

[11] Sparrow, E. M. and Cess, R.D., The effect of a magnetic field on free convection heat transfer, Int. J. Heat and Mass Transfer, Vol. 3, (1961), pp 267-274.

[12] Lai, F. C. and Kulacki, F.A., The effect of variable viscosity on convective heat and mass transfer along a vertical surface in saturated porous media, Int. J. of Heat and Mass trasfer, Vol. 33 (1991), pp 1028-1031.

[13] Ling, J. X. and Dybbs, A., Forced convection over a flat plate submerged in a porous medium: variable viscosity case, paper 87, Wa/HT-23, (1987), ASME. 9

Authors' address
G. C. Hazarika — Department of Mathematics, Dibrugarh University, Dibrugarh, India
e-mail: gchazarika@gmail.com
Utpal Sarma Gopal Ch. — Joya Gogoi College, Khumtai, India
e-mail: utpalsarmagjc@rediffmail.com